



Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

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Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

Valentin Resseguier,
Etienne Mémin,
Bertrand Chapron



Motivations

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Studying bifurcations and attractors



Climate projections

- Quantification of modeling errors



Ensemble forecasts and data assimilation

Contents

- Scalian
- Location uncertainty
- SQG under moderate uncertainty

Part I : SCALIAN

L@b
(~ 15 peoples)

Research, R&T, R&D

Expertise:

- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

CEN « Simulation » (~ 70 people)

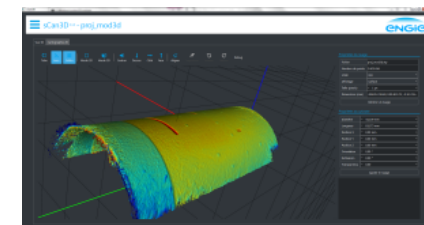
R&D and engineering

Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:

- Scientific softwares
- Simulations, HPC
- VR & AR



Other Business Units

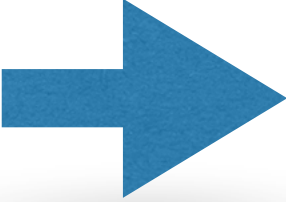
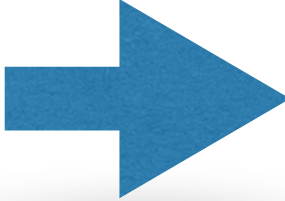
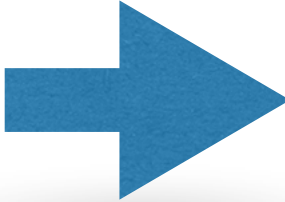
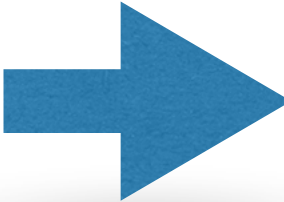
~ 2400 people



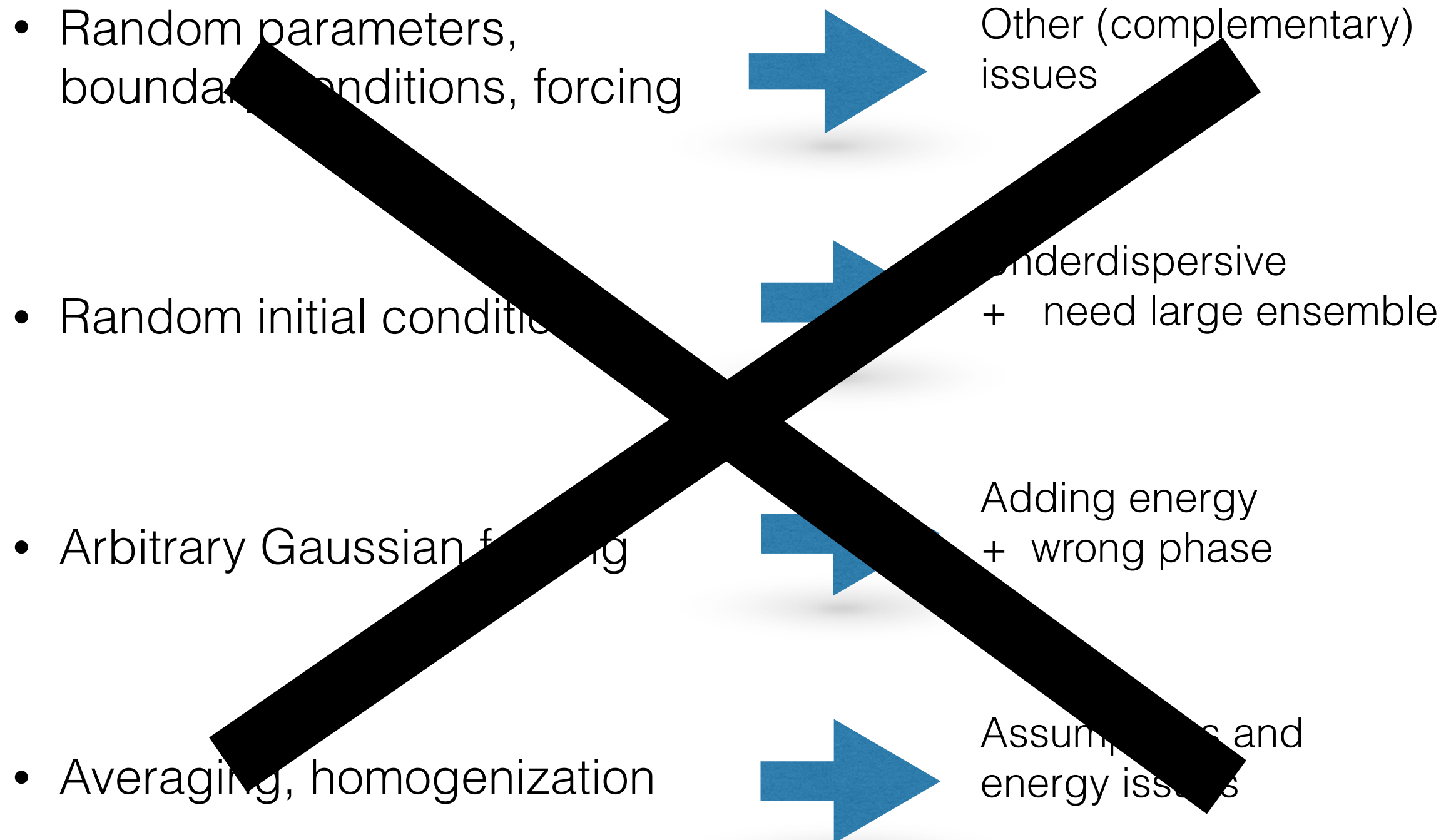
Part II

Location uncertainty (LU)

Usual random CFD

- Random parameters, boundary conditions, forcing  Other (complementary) issues
- Random initial conditions  Underdispersive + need large ensemble
- Arbitrary Gaussian forcing  Adding energy + wrong phase
- Averaging, homogenization  Assumptions and energy issues

Usual random CFD

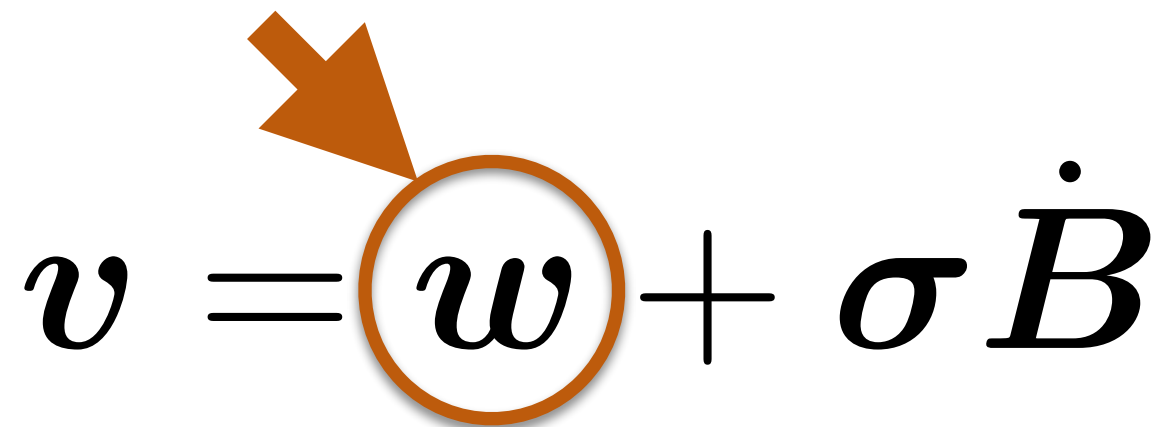


LU : Adding
random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

LU : Adding random velocity

Resolved
large scales



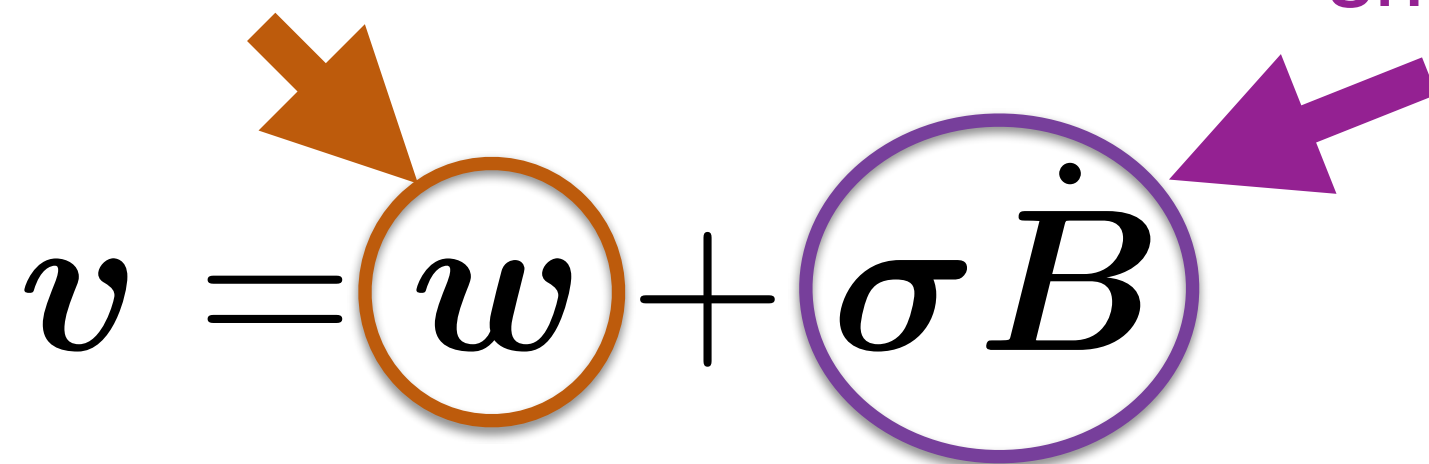
The diagram shows the equation $v = w + \sigma \dot{B}$. The variable w is enclosed in a brown circle, and a brown arrow points from the text "Resolved large scales" to this circle.

$$v = w + \sigma \dot{B}$$

LU : Adding random velocity

Resolved
large scales

White-in-time
small scales



The diagram shows the equation $v = w + \sigma \dot{B}$. The term w is enclosed in an orange circle, and an orange arrow points from the text "Resolved large scales" to it. The term $\sigma \dot{B}$ is enclosed in a purple circle, and a purple arrow points from the text "White-in-time small scales" to it.

$$v = w + \sigma \dot{B}$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

LU : Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

LU : Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius and
Rozovskii, 2004
Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Chapron et al. 2017
Cai et al. 2017

Holm, 2015
Holm and
Tyranowski, 2016
Arnaudon et al., 2017

Cotter and al 2017
Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al 2018 a, b

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \boxed{\text{Advection}} \quad w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing

$$\partial_t \Theta + \underbrace{w^\star \cdot \nabla \Theta}_{\text{Advection}} + \sigma \dot{B} \cdot \nabla \Theta = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing

$$\partial_t \Theta + \underbrace{w^\star \cdot \nabla \Theta}_{\text{Advection}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Diffusion}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Drift correction

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Drift correction

Advection

Diffusion

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

Advection of tracer Θ

Multiplicative
random
forcing

Balanced
energy
exchanges

Advection

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Diffusion

Drift correction

A word about reduced order models

- Very fast simulation of very complex system (e.g. for industrial application)
- Physical model (PDE) simplified using observations

Reduced models under location uncertainty

- Rigorous and low-cost estimators
- Stabilization of the unstable modes
- Maintain variability of stable modes
- Uncertainty quantification

State of art

- Possible parametrization with eddy viscosity
- Impossible to parametrize with additional dissipation
Need ad hoc closure like MQG
(Sapsis and Majda, 2013a,b,c)

Part III

SQG under Moderate Uncertainty

SQG MU

Code available online

$t = 17 \text{ days}$

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

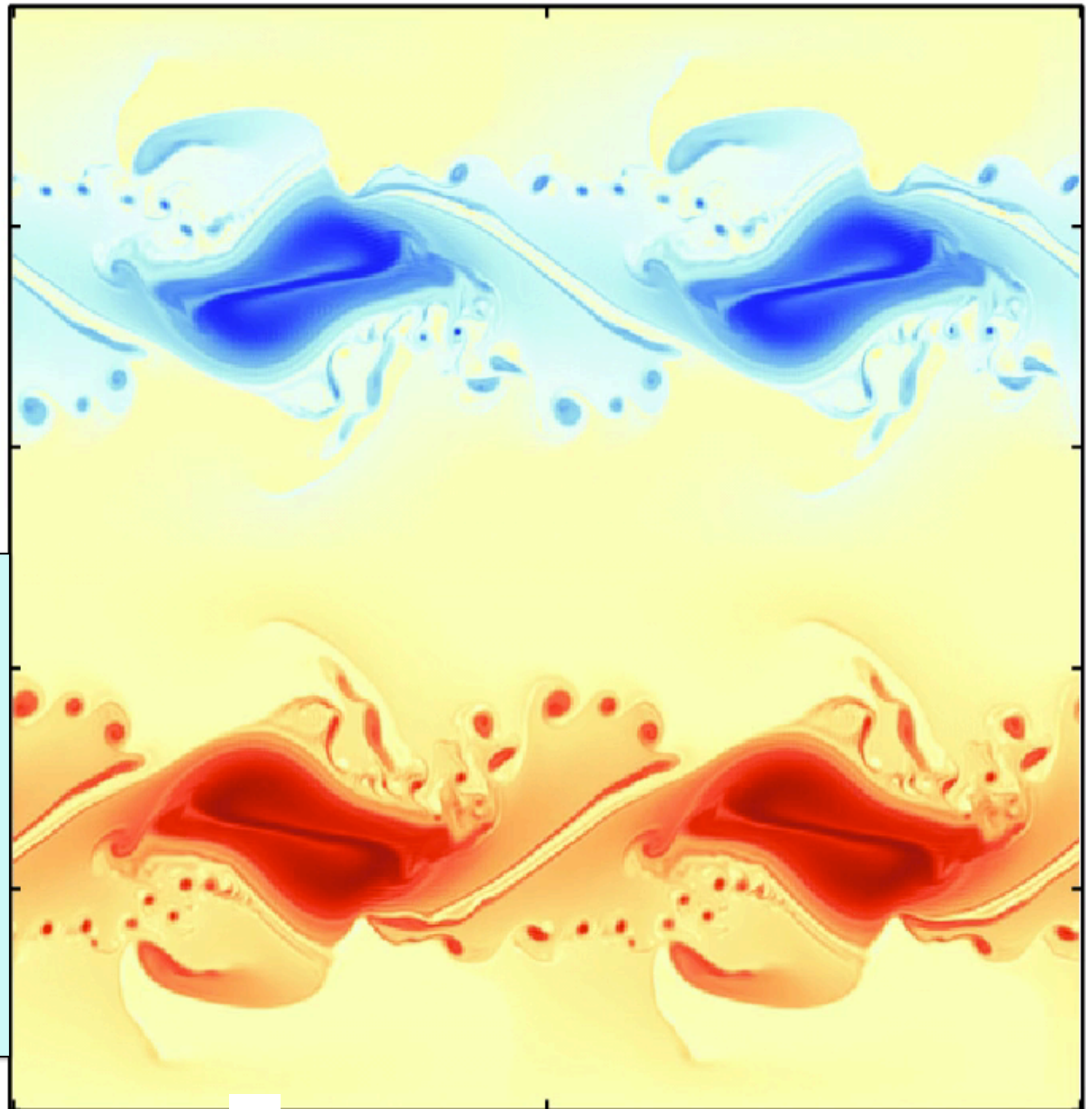
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

1024 x 1024



$t = 17 \text{ days}$

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

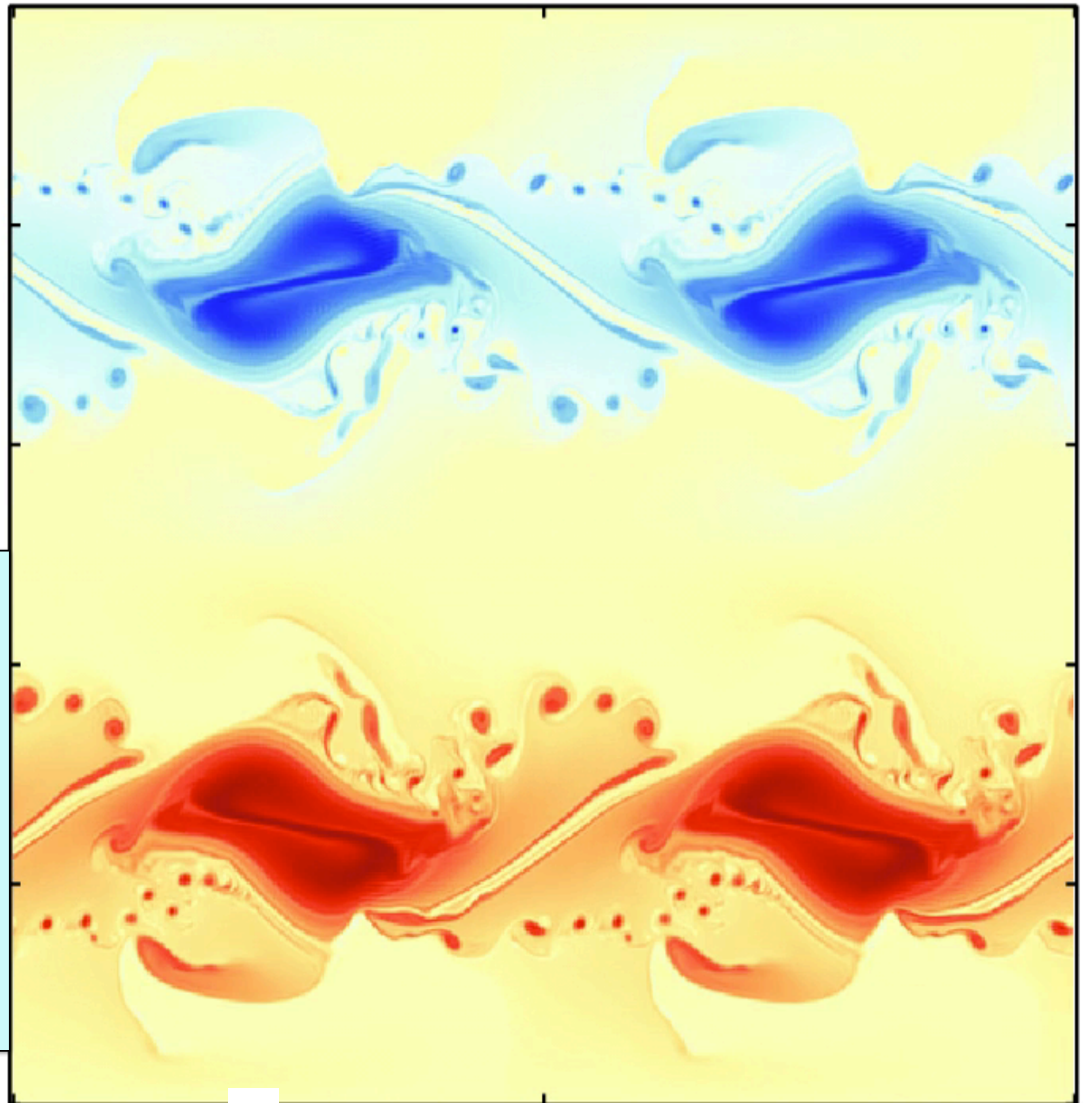
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

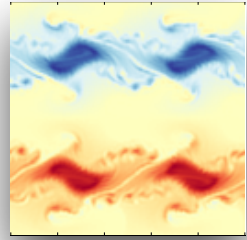
Reference flow:

deterministic

SQG

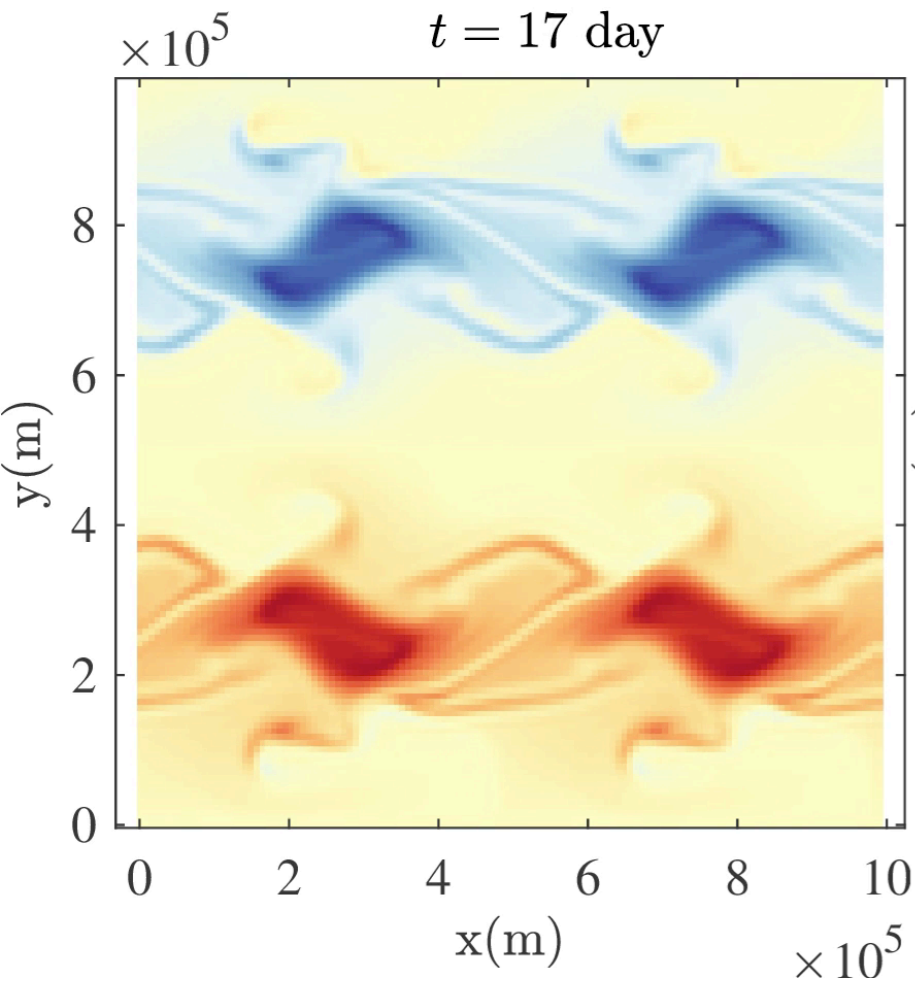
1024 x 1024



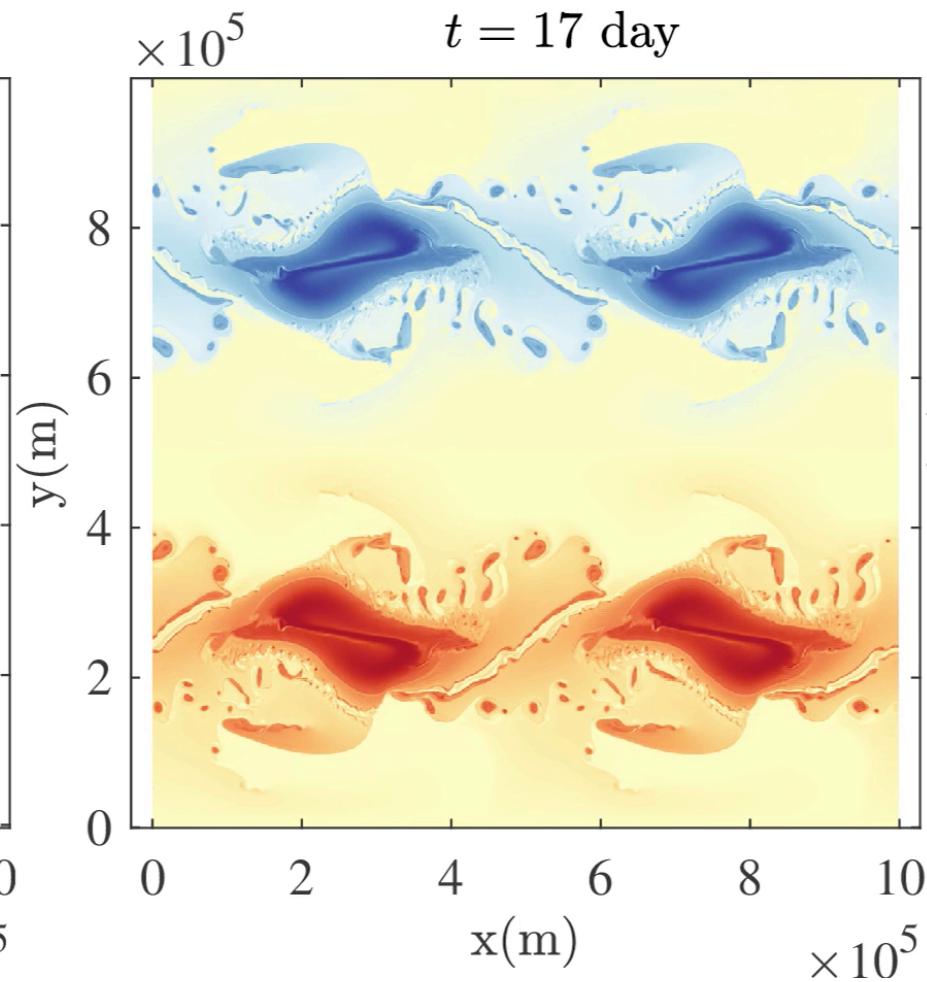


One realization : Stochastic destabilization

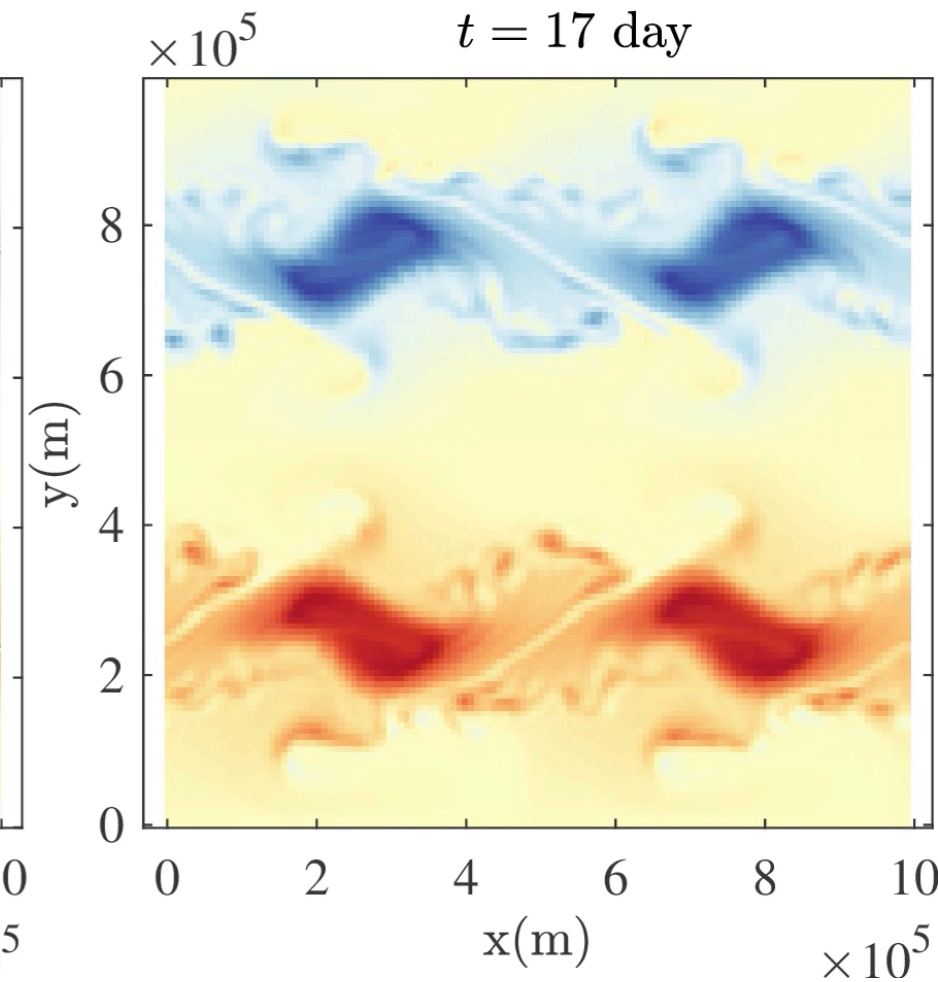
Deterministic 128 x 128

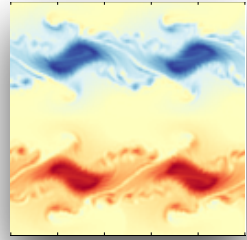


Deterministic 1024 x 1024



Location Uncertainty 128 x 128



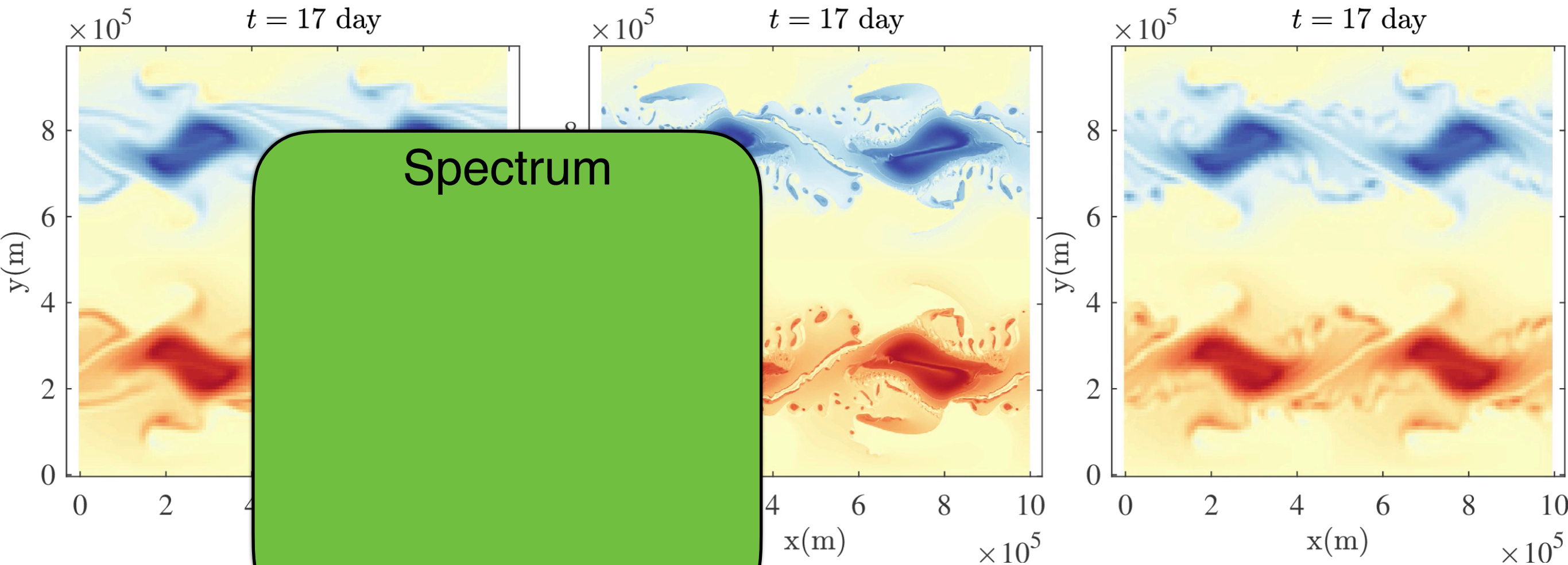


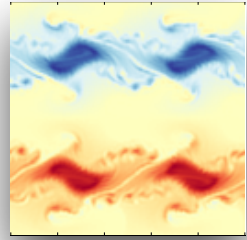
One realization : Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128



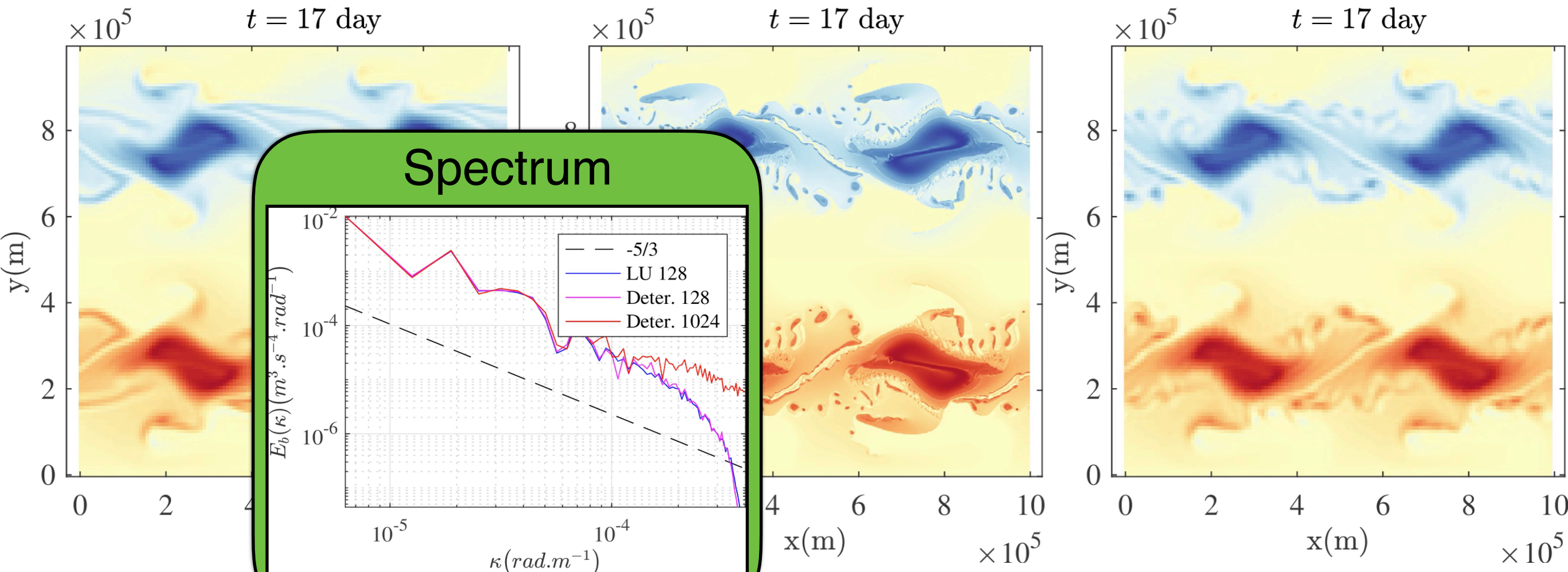


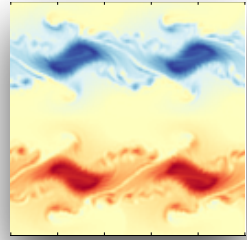
One realization : Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128



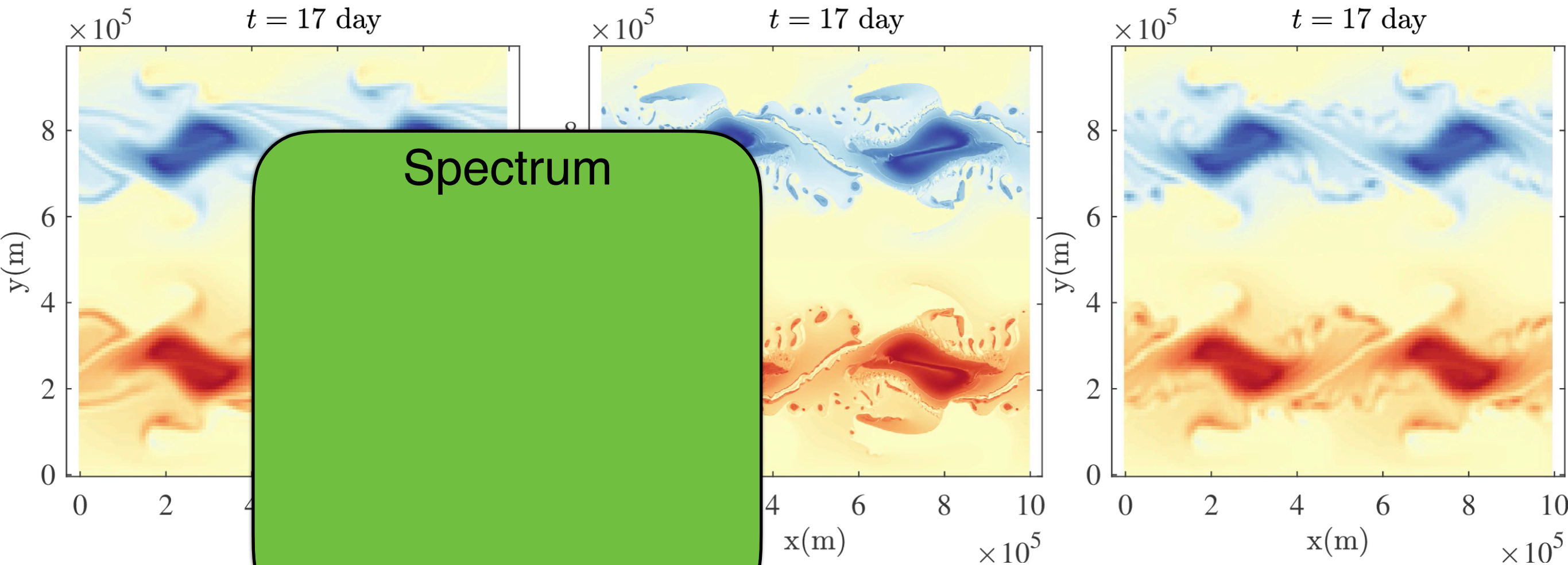


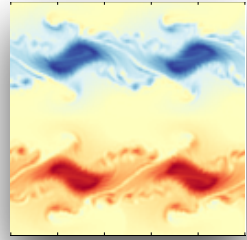
One realization : Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

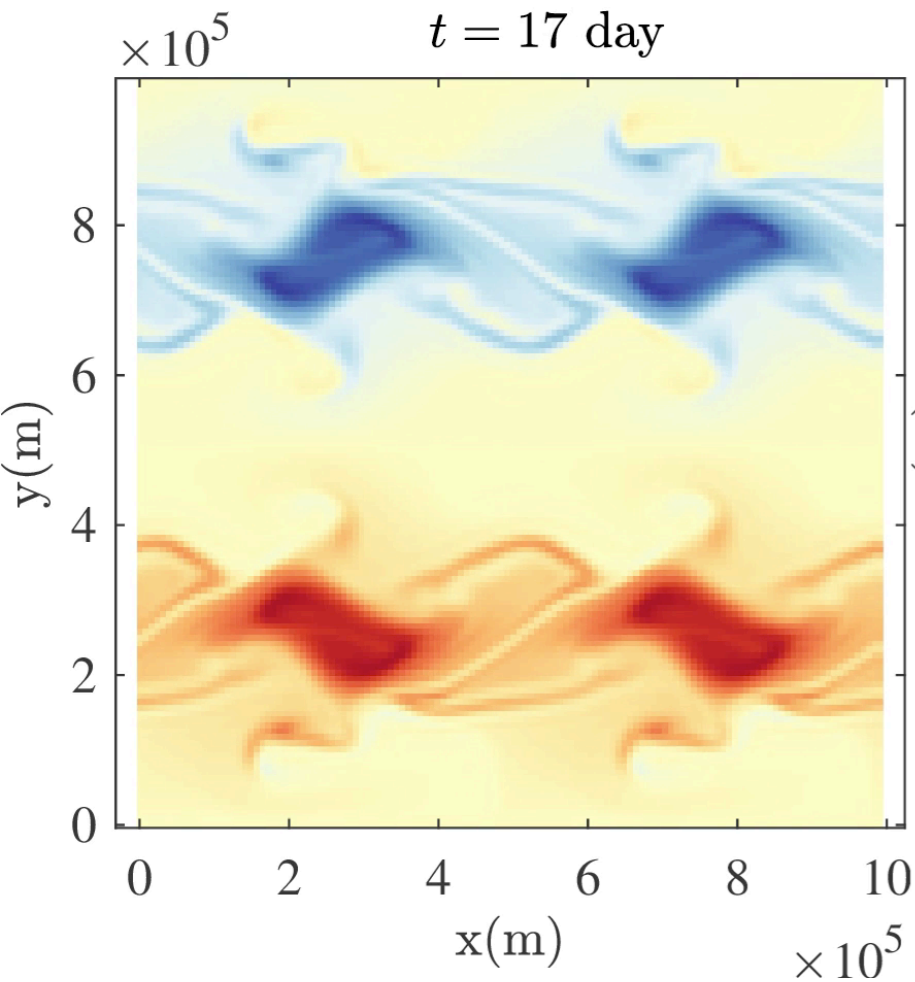
Location Uncertainty 128 x 128



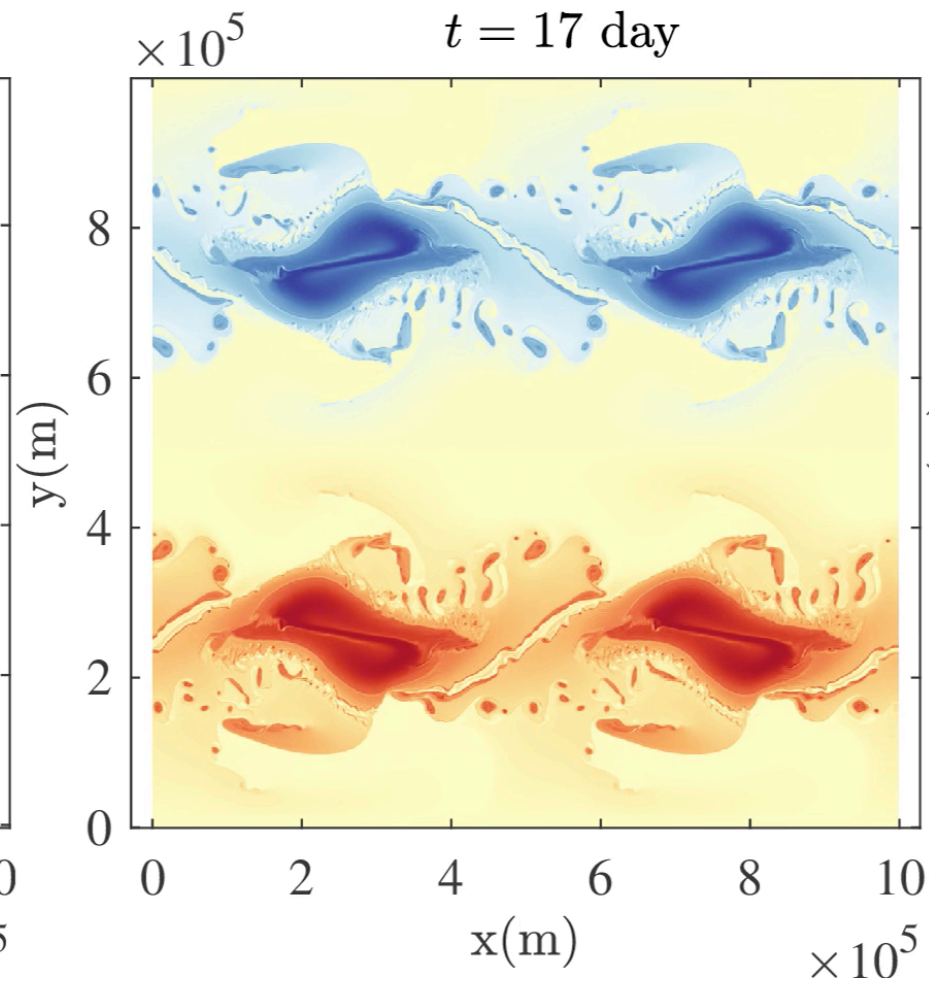


One realization : Stochastic destabilization

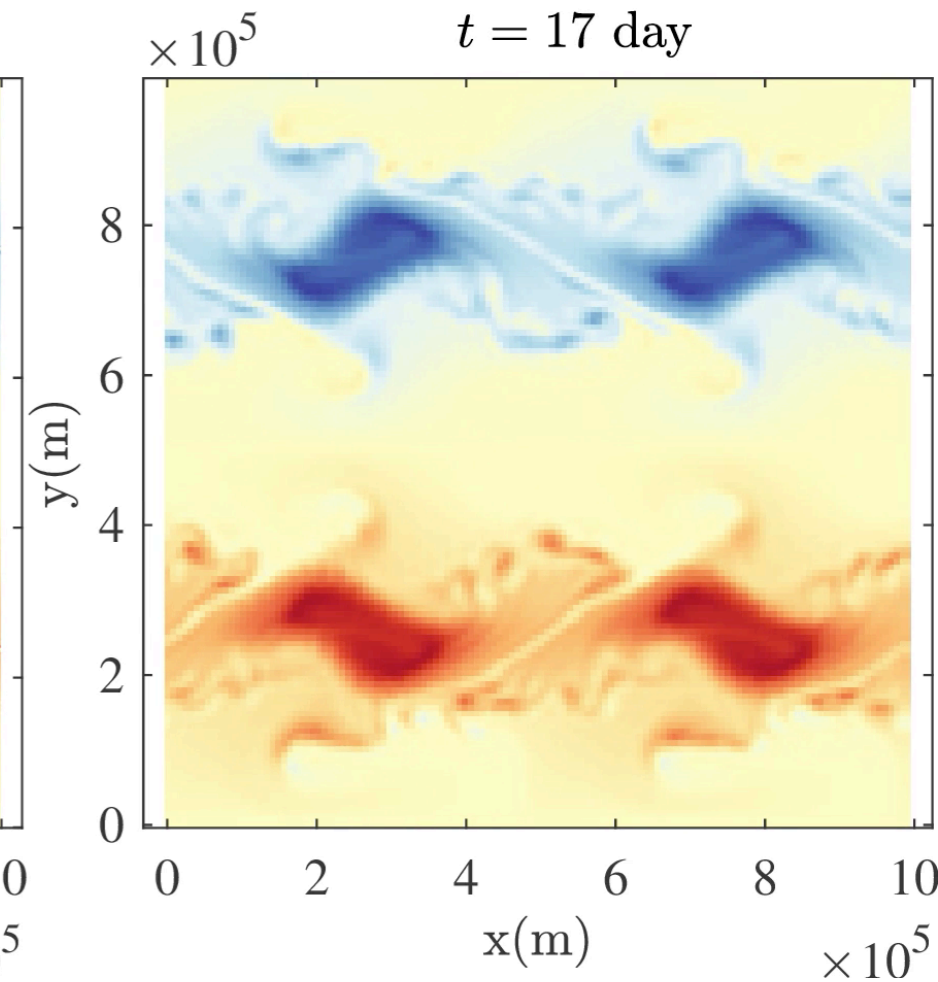
Deterministic 128 x 128

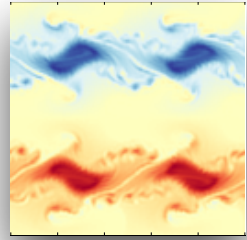


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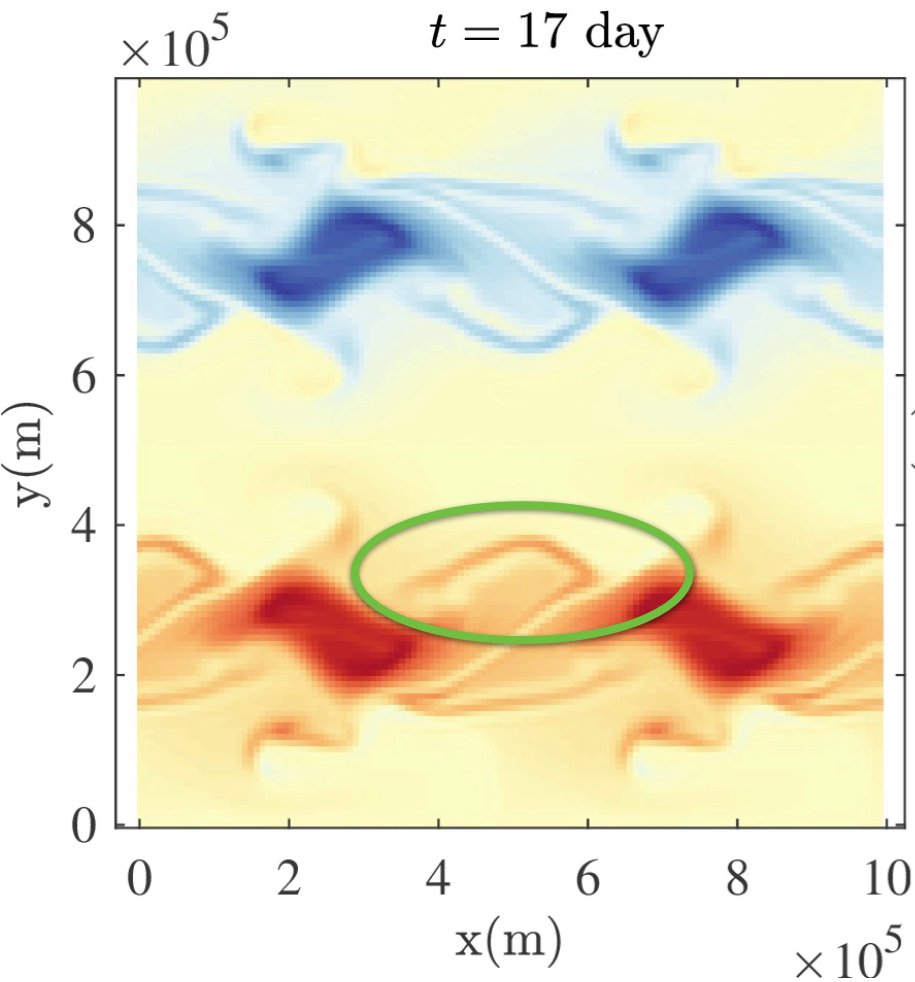
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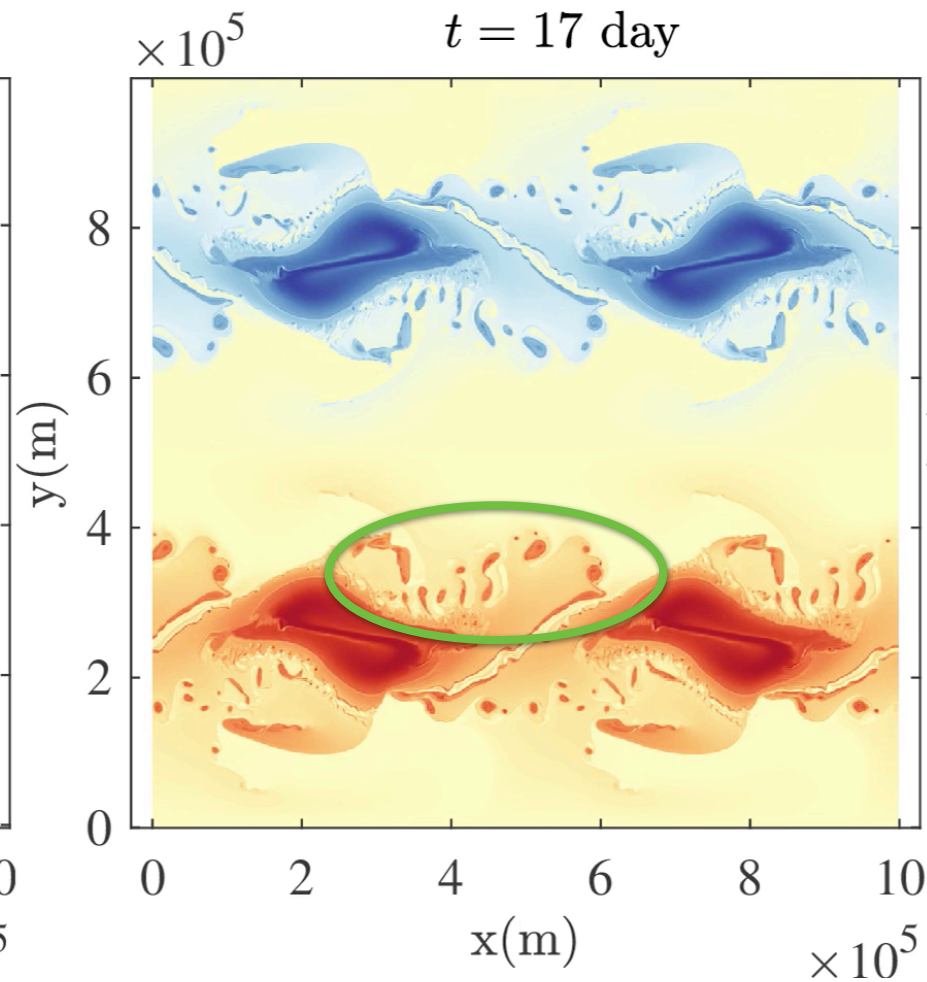


One realization : Stochastic destabilization

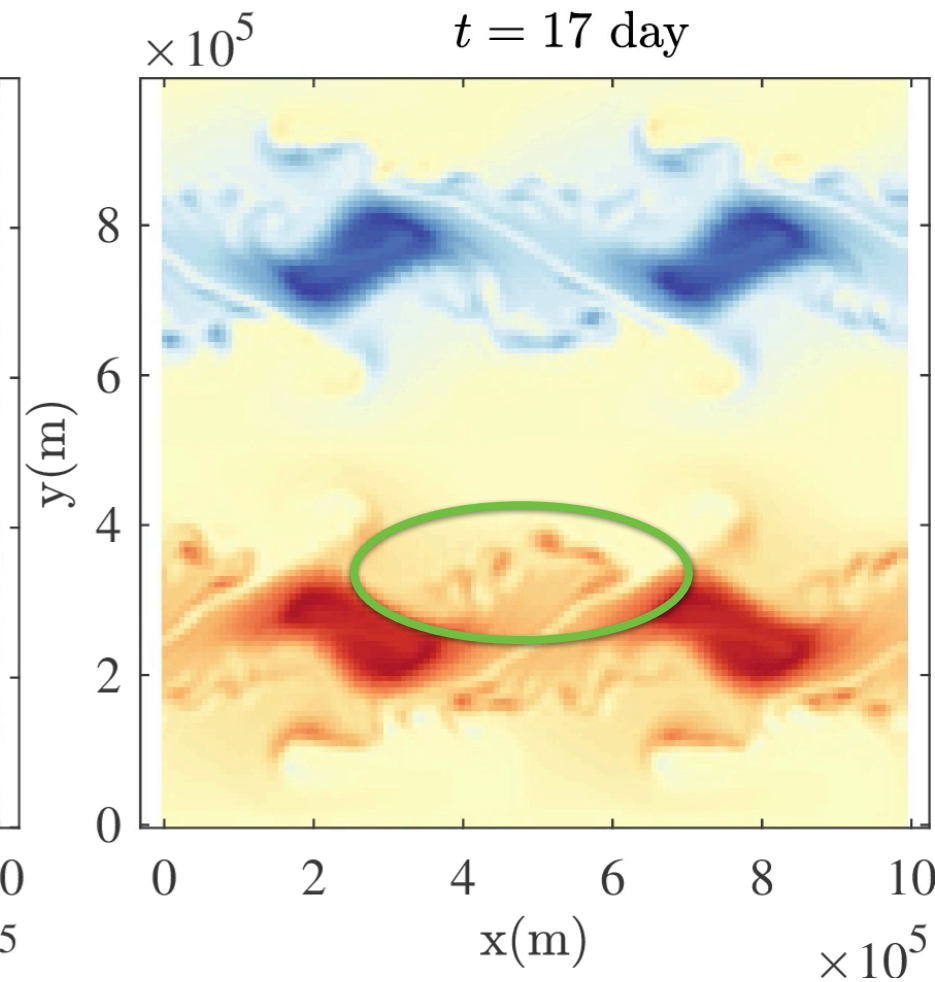
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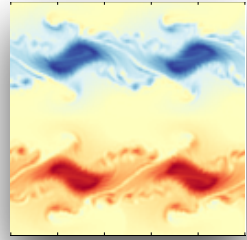


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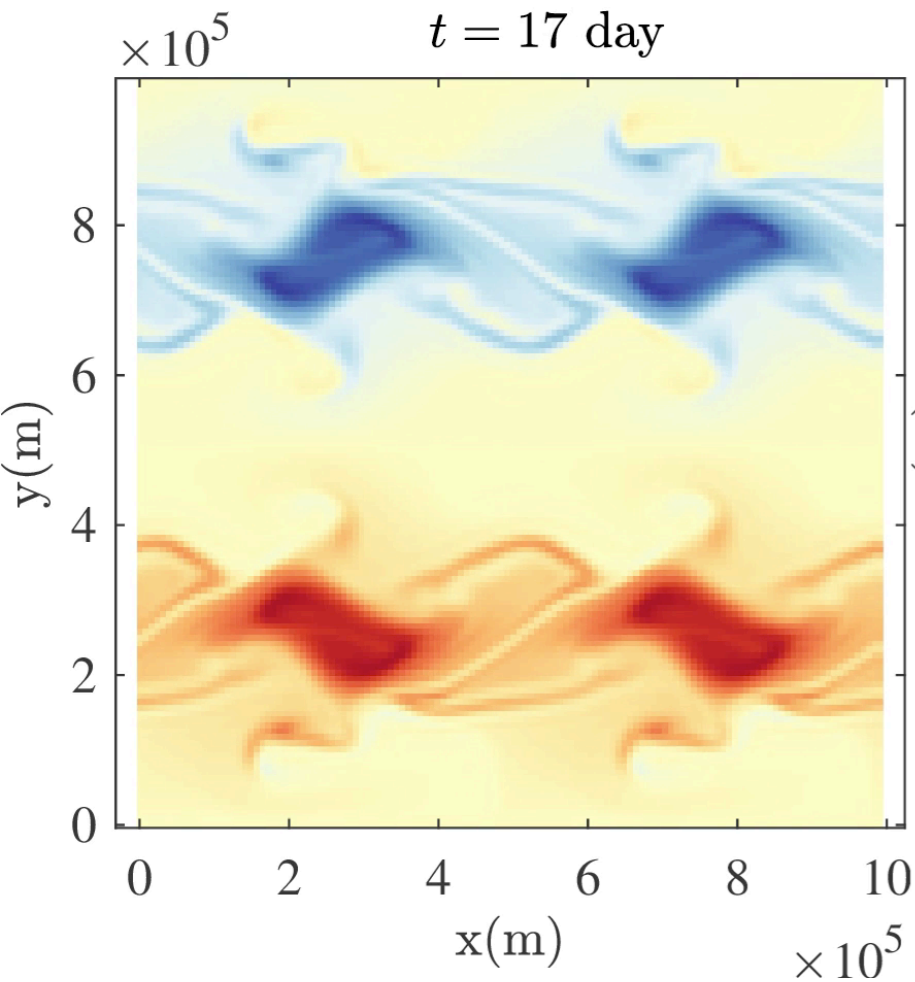
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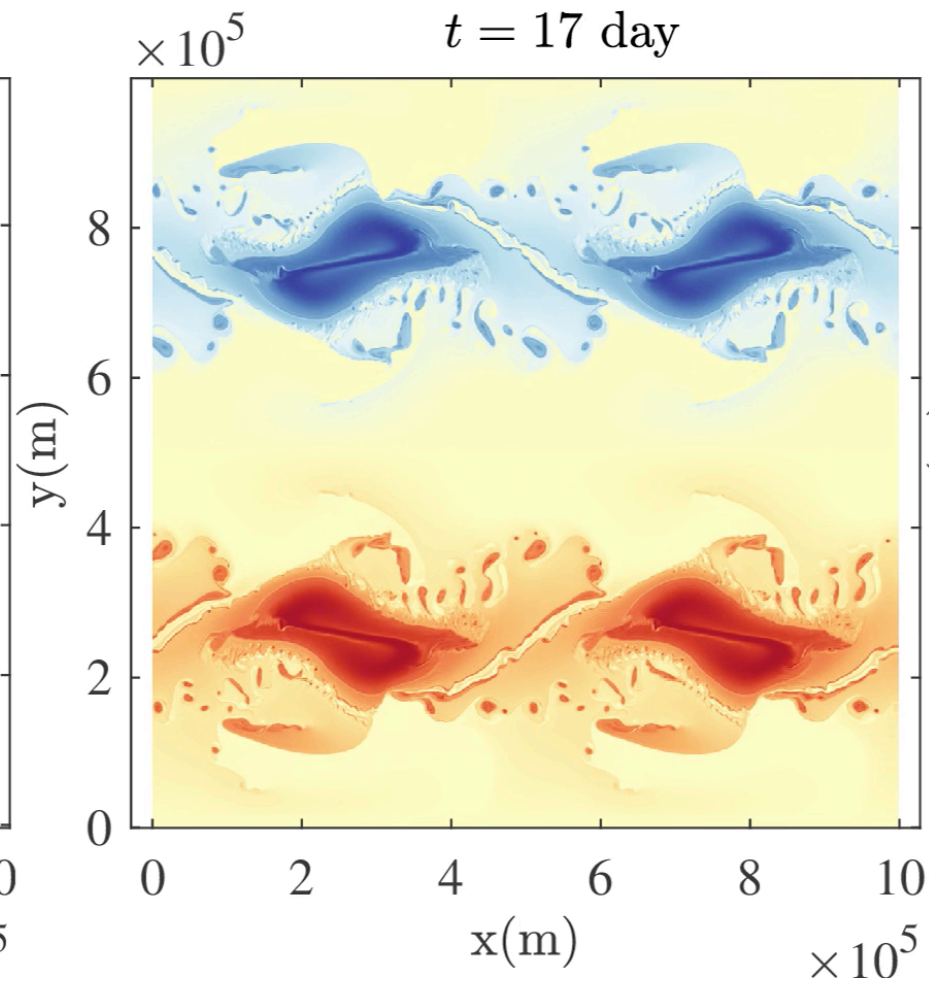


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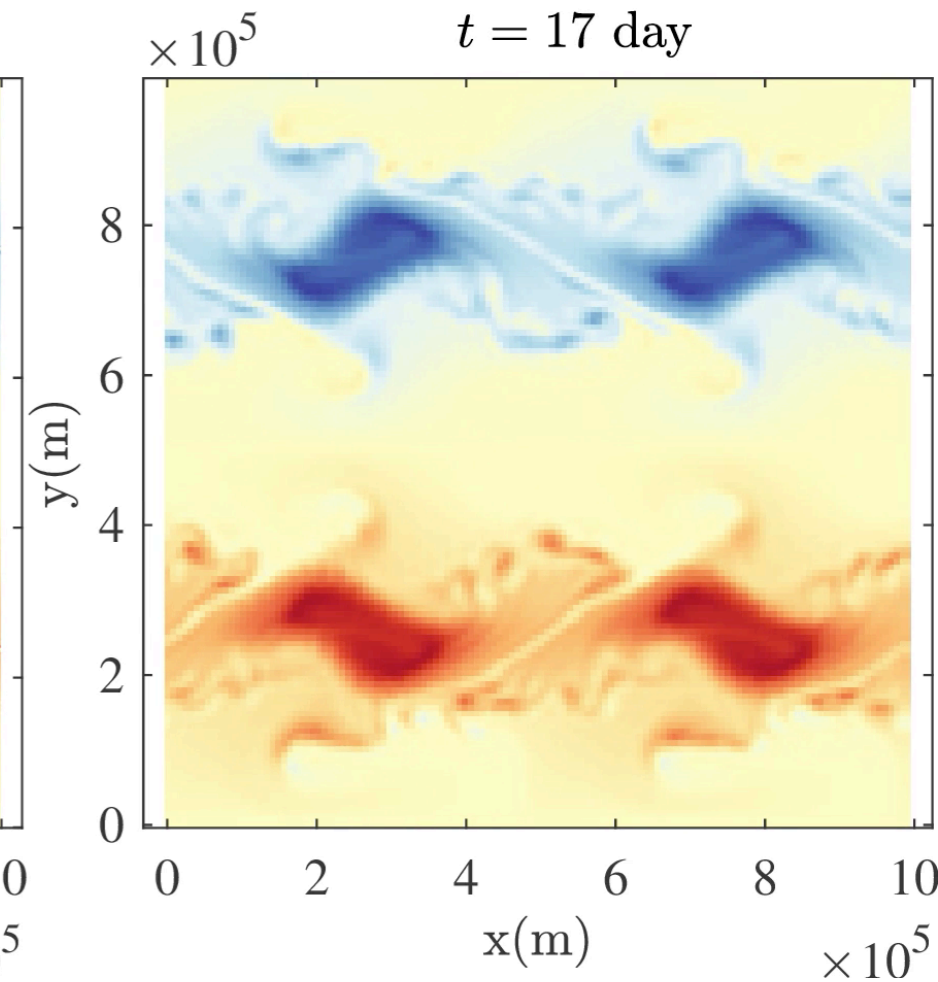
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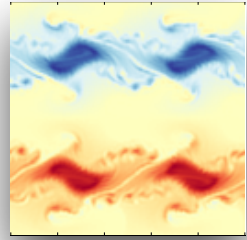


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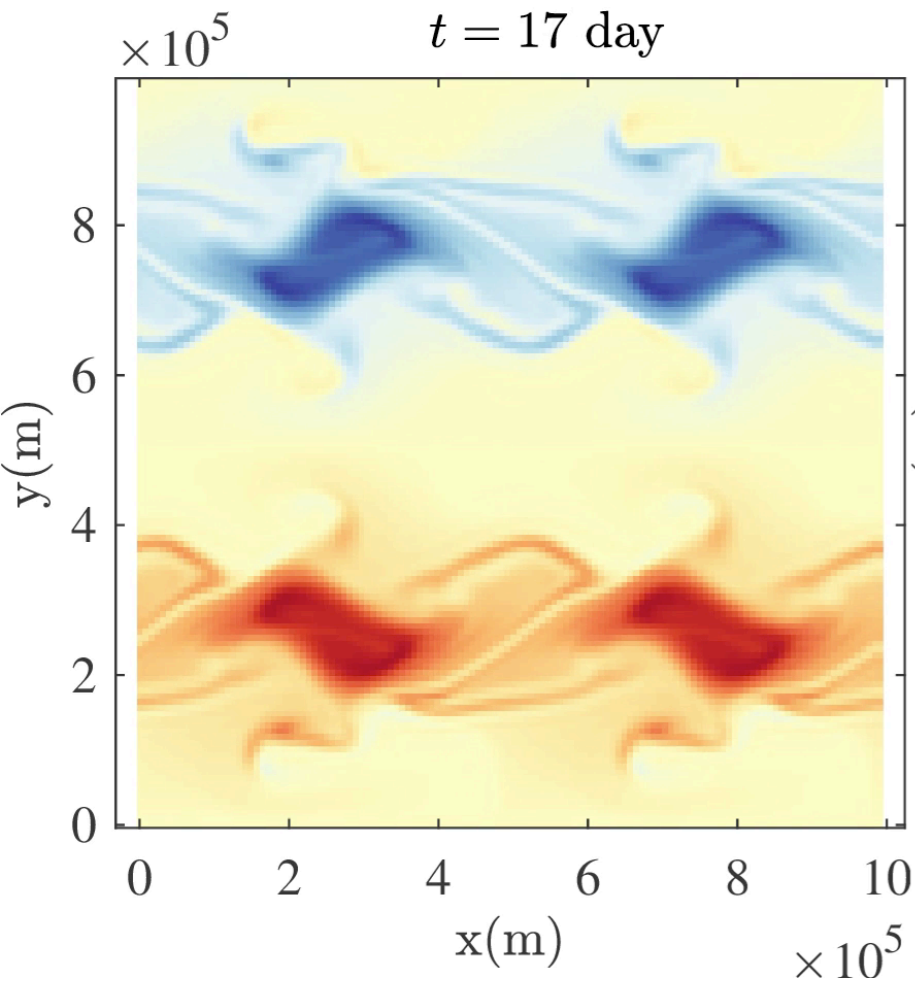
Location Uncertainty 128 x 128



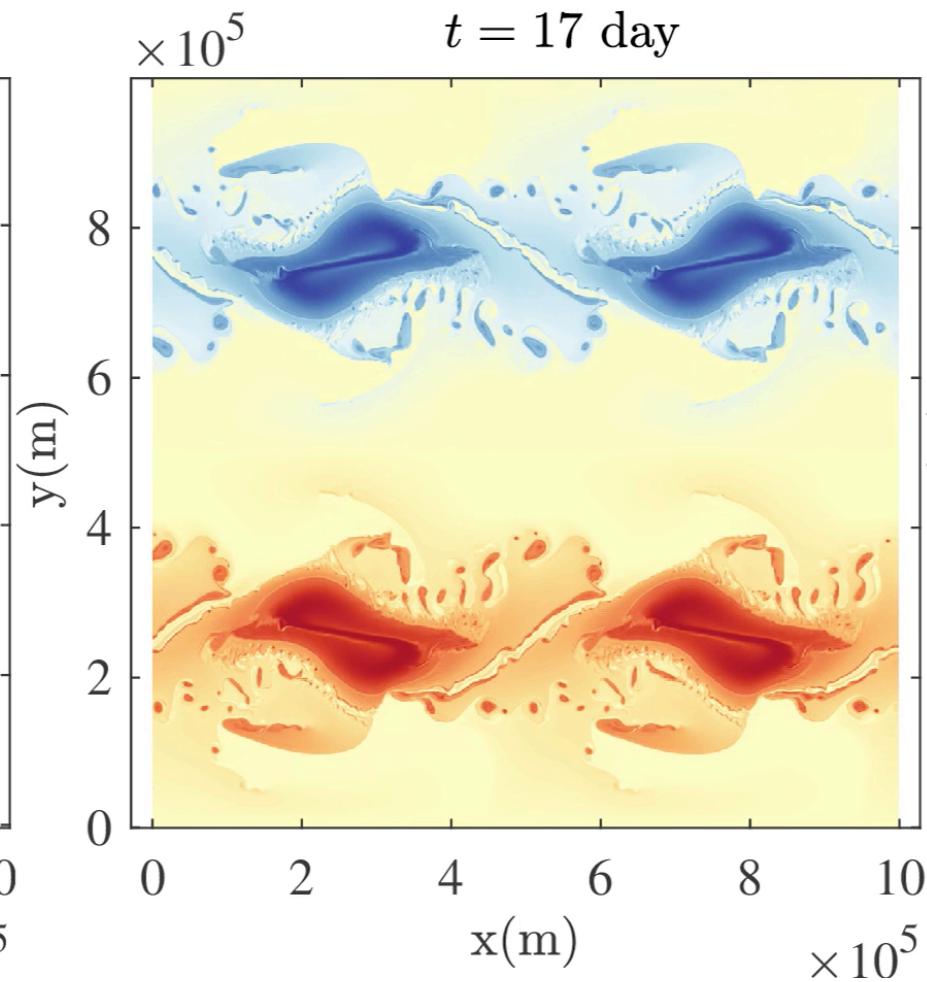


One realization : Stochastic destabilization

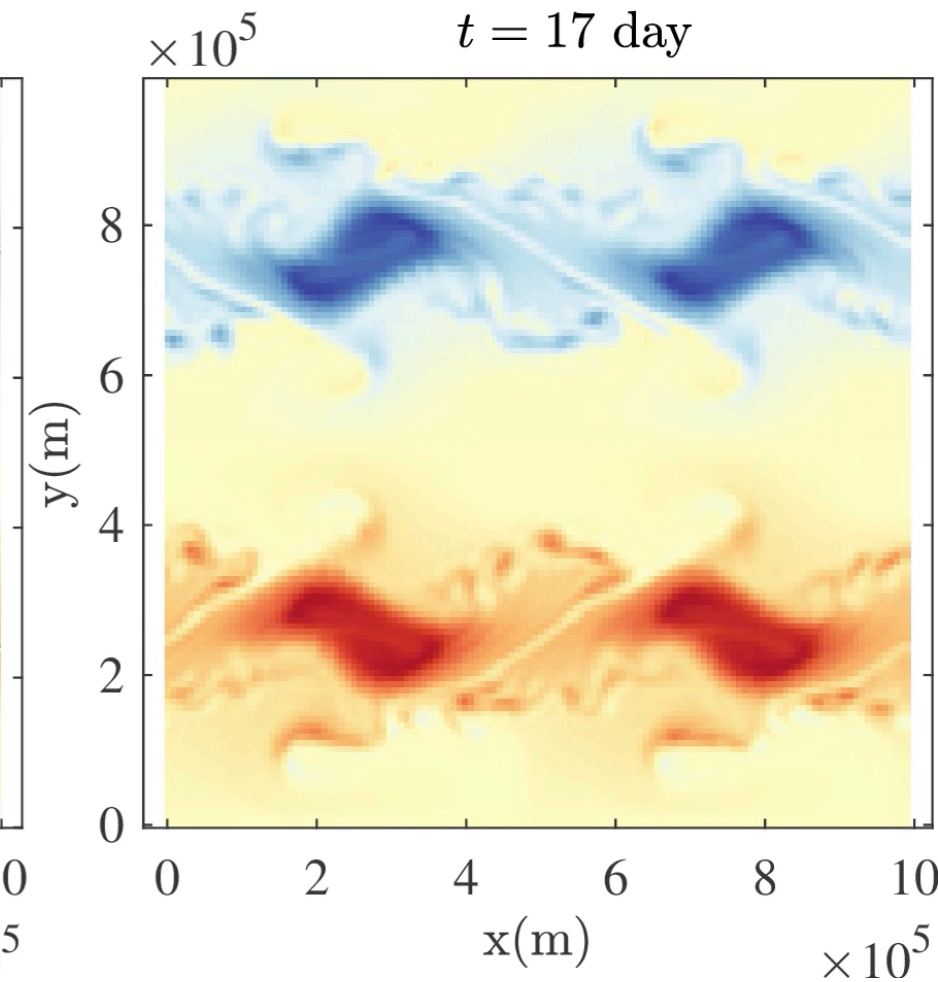
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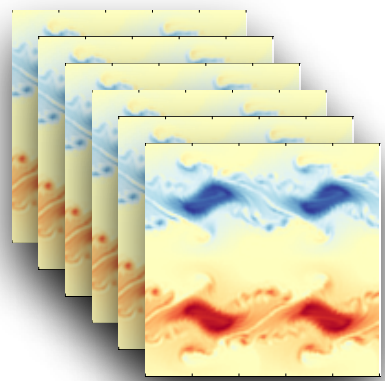


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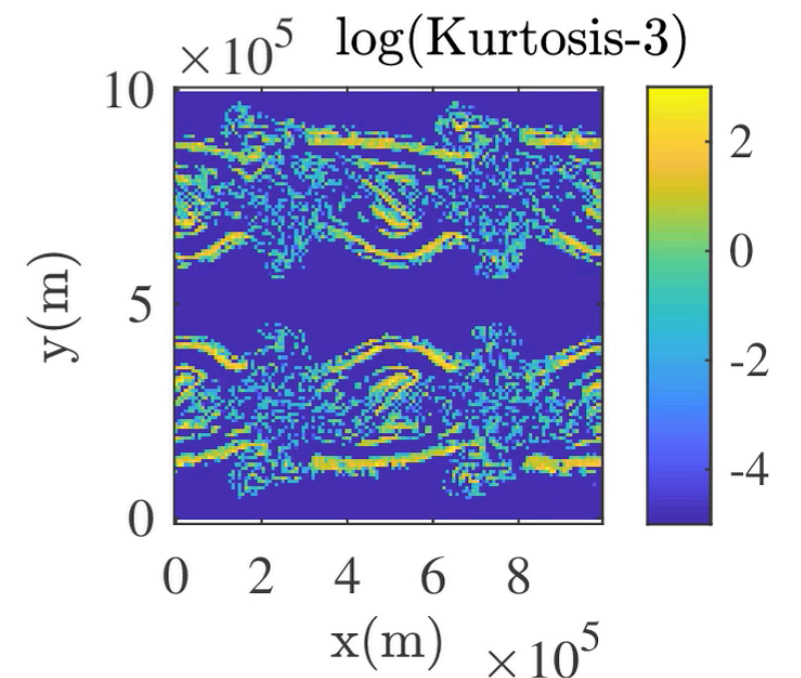
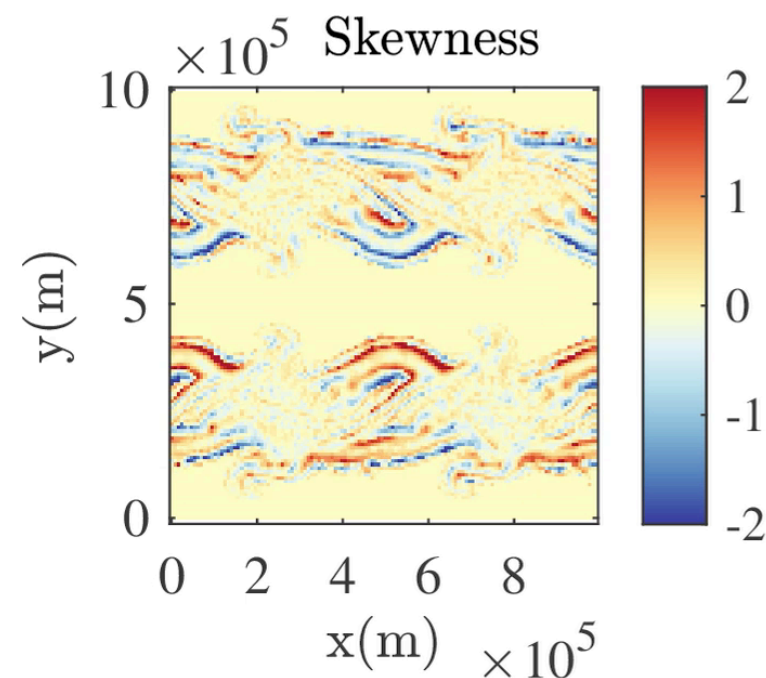
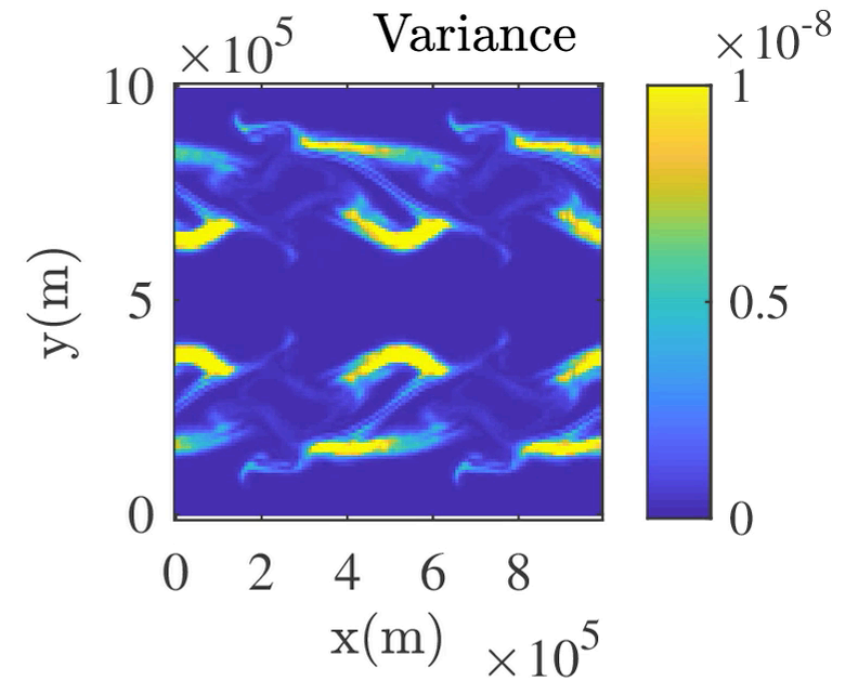
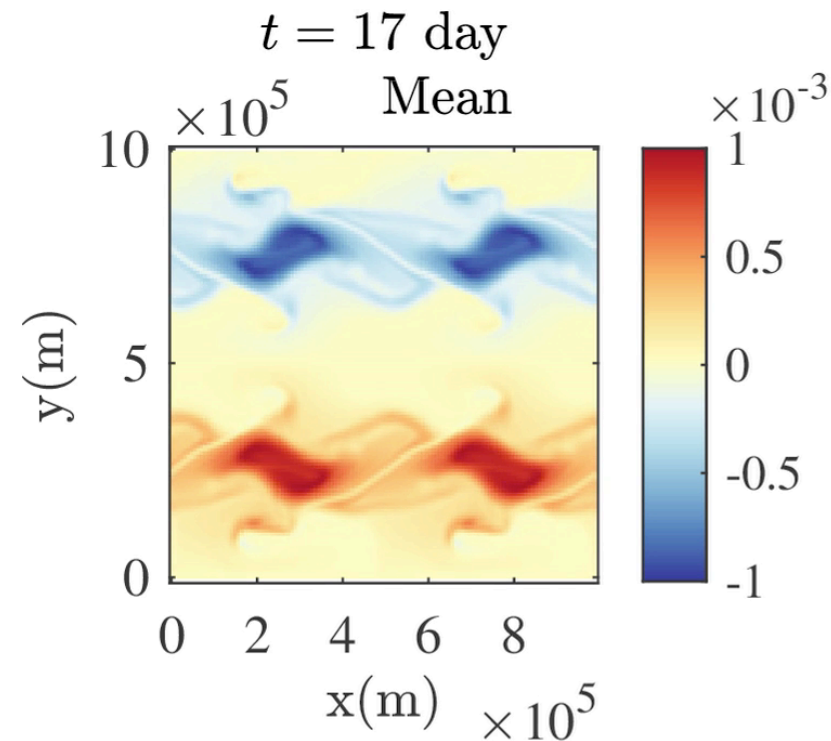


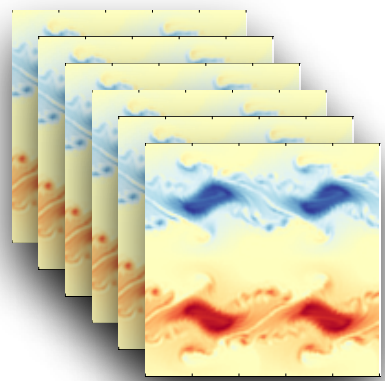
Location Uncertainty 128 x 128



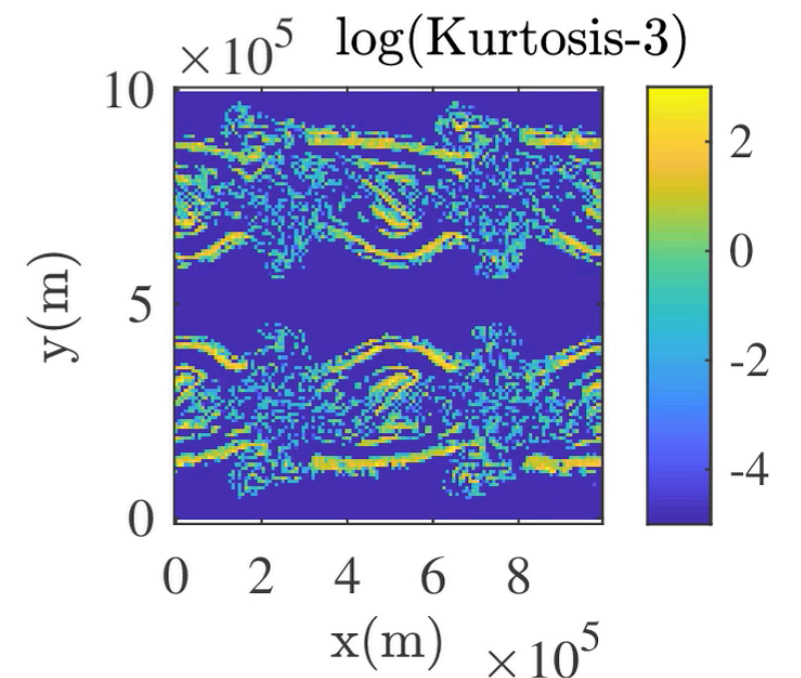
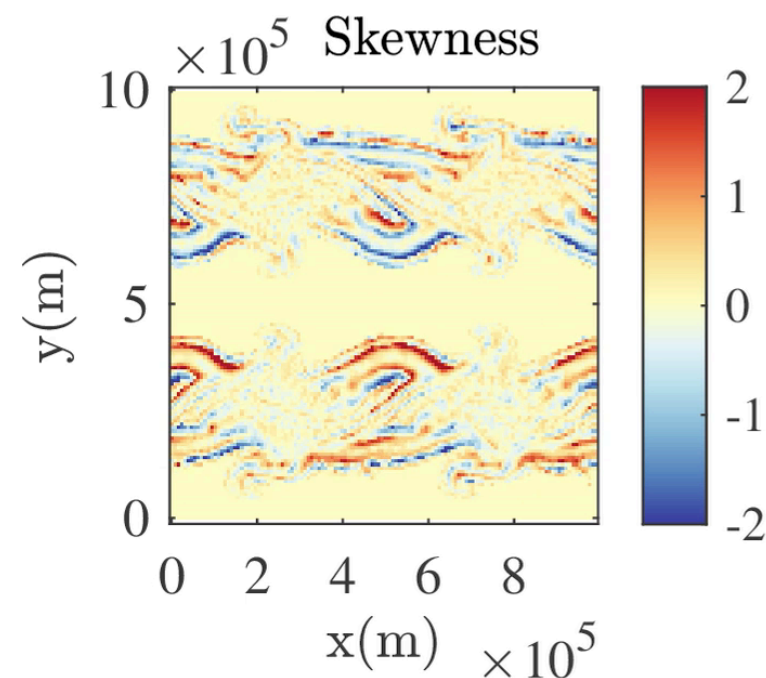
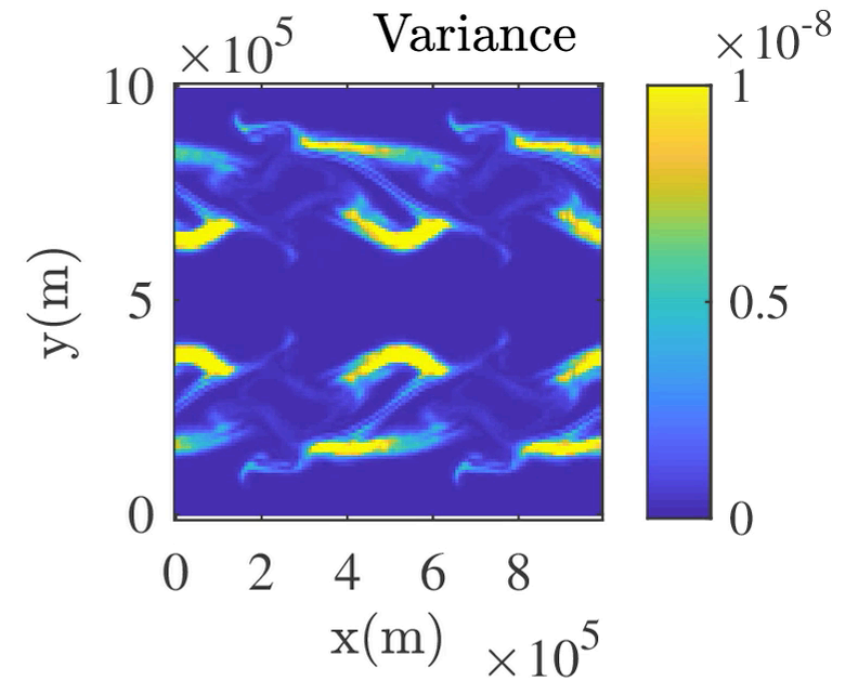
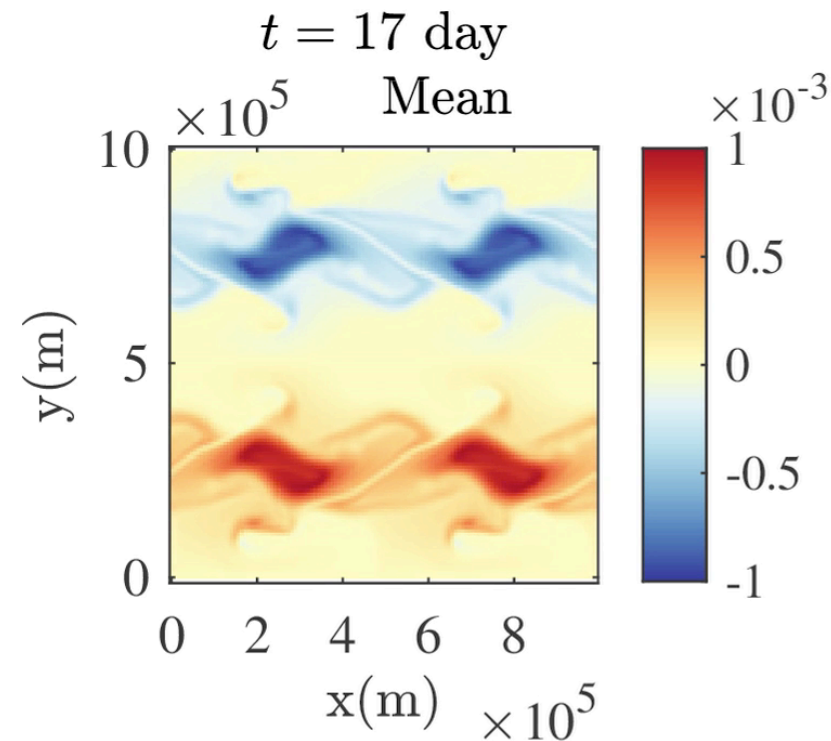


Ensemble :
random
coherent
structures

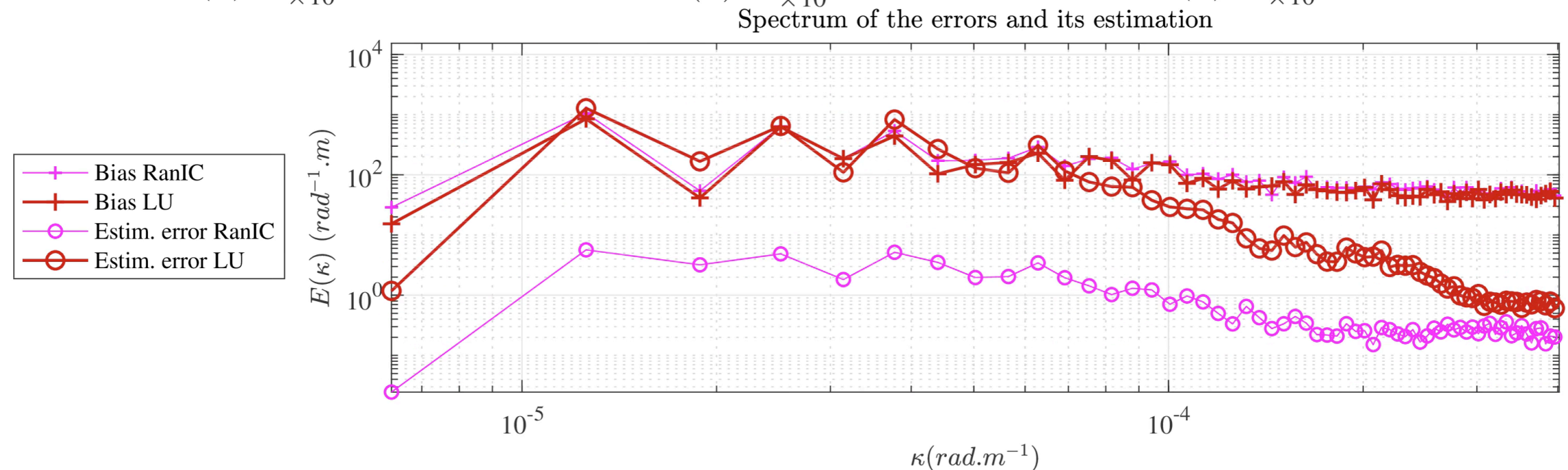
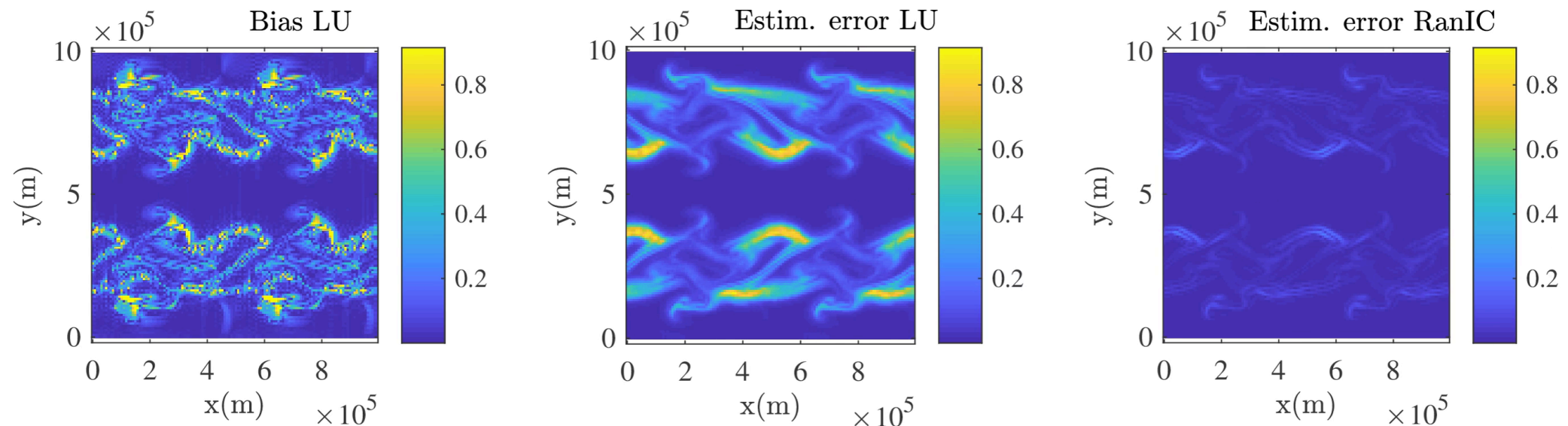




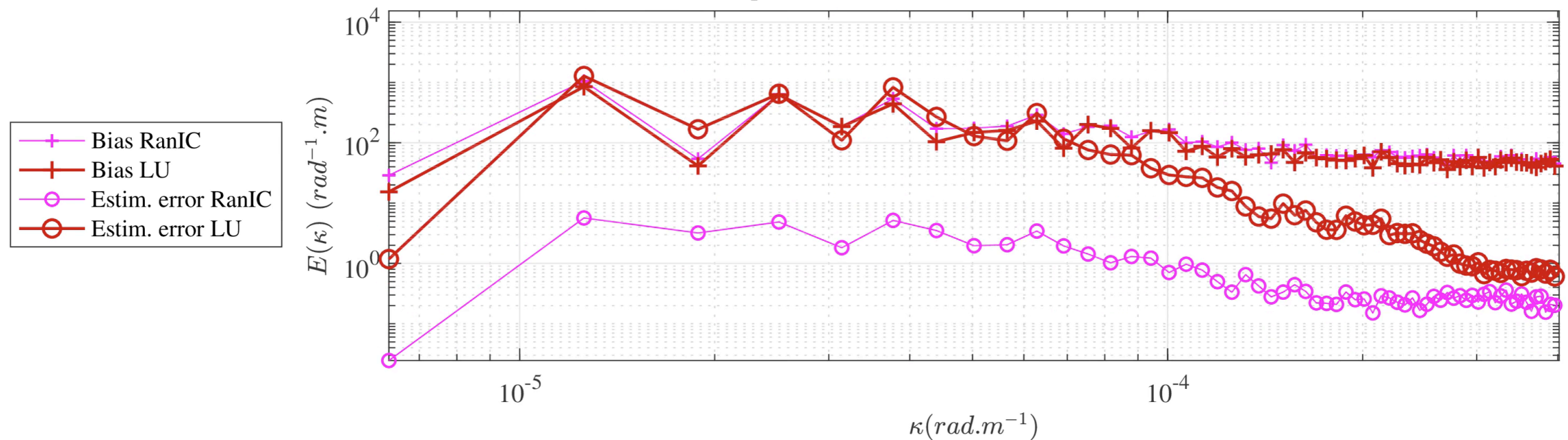
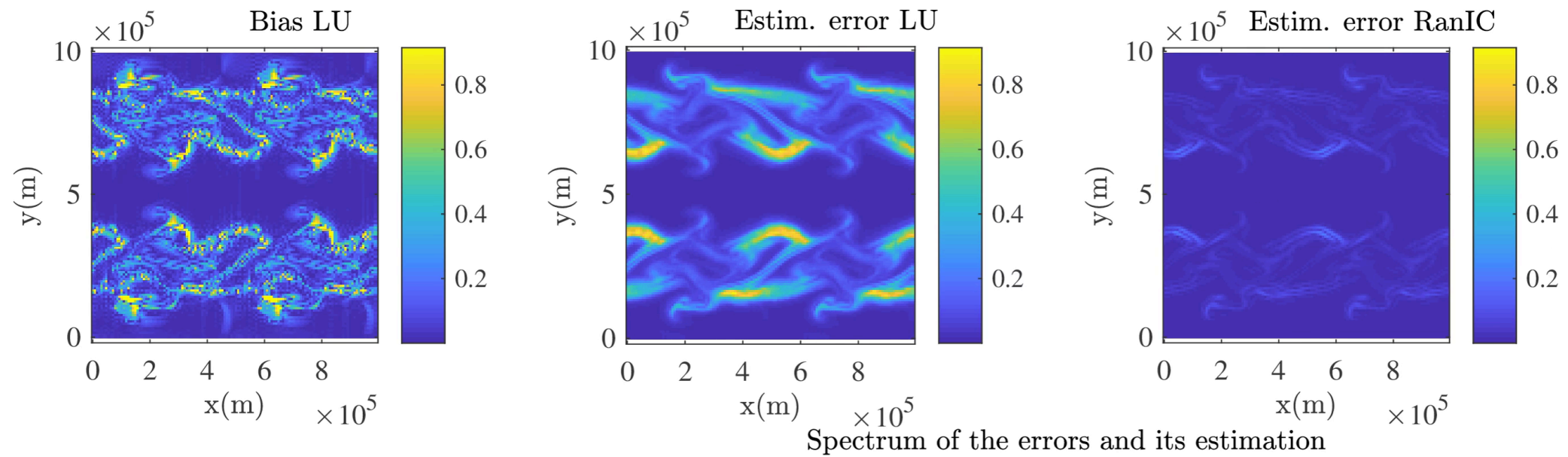
Ensemble :
random
coherent
structures



Ensemble : uncertainty quantification



Ensemble : uncertainty quantification



Conclusion

Conclusion

LU models blindly describe unresolved triades

- Conserve energy
- Stabilization / destabilization in Reduced Order Model
- Instabilities triggered, possibly followed by extreme events
- Uncertainty quantification to address filter divergence

Related works, outlooks and application

- Bifurcations (SQG) and attractor (Lorenz 63) exploration
- Comparisons with data-driven parametrisation and SALT (Stochastic Advection by Lie Transport) (Holm and coauthors)
- Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, ...)
- (Surface gravity) wave / turbulence interaction
- **Data assimilation (DA) :**
 - **Filtering / smoothing**
 - **EnKF with LU model as a R&D tool** (for e.g. airplanes, drones)
 - **PF with reduced LU model for real-time monitoring and flow control** (for e.g. pollutant dispersion monitoring, drag and damage reduction in e.g. wind turbines)
 - Girsanov theorem for MLE and Bayesian estimations with e.g. satellite images